

A REVIEW OF THE DEFINITIONS OF THE TERM “MATHEMATICS”

Elena Hadzieva

University of Information Science and Technology “St. Paul the Apostle”-
Ohrid, RN Macedonia

Abstract

The purpose of definitions in mathematics is to ensure the precise meaning of mathematical notions and enable clear and rigorous communication of mathematical concepts. However, due to the complexity of the term “mathematics”, scientists and linguists haven’t agreed on its definition. A variety of definitions is offered throughout the literature, formed from different perspectives. They reflect the understanding of mathematics at a particular time in history, by a particular community, and often by a single person. In this paper, 50 definitions are collected, classified, and analyzed from the perspective of a mathematician. We consider that the collection exposed here will be inspiring for other researchers, and further analysis from different viewpoints will be elaborated.

Keywords: mathematics, definition, dictionary definition, working definition.

INTRODUCTION

Definitions are an essential part of every formal research as they set up the knowledge base, provide means to reason about concepts, to connect and compare facts. In the most general sense, a definition describes the meaning and usage of a specified term. Formally, a definition consists of the definiendum (that which is to be defined), the definiens (that which defines), and the defining connective [13]. E.g. in the following definition, “An equation is a mathematical expression that contains an equals symbol”, “an equation” is the definiendum, “a mathematical expression that contains an equals symbol” is the definiens, and “is” is a defining connective. From a linguistical aspect, the definition must be linguistically congruent (e.g. plural noun shouldn’t be defined by a singular noun, a noun should not be defined by a verb phrase), categorically congruent (definiens and definiendum should belong to the same general category of concepts), and congruent in variables (the definiens and definiendum should use the same variables) (see [13] for a more detailed explanation).

The definitions in mathematics are typical for their rigor and usually satisfy all these properties. They include exactly what they should cover, nothing less, nothing more (except some terms which are defined for general addresses or youngest students).

Contemporary mathematical norms dictate that once a definition is agreed upon, it is essentially indisputable ([2]). With the term mathematics, the contrapositive of the last implications explains the situation: its definition is disputable, thus there is no consensus for a single definition that the mathematical learners and practitioners will be content with. The non-existence of the definition is noted in the German and English Wikipedia, as well as in [16] and [40]. There exist only definitions, or better to say, descriptions of “mathematics”, that are proper in a specific context. All of them give a glimpse to the reader of what mathematics is and the number of aspects mathematics can be viewed from.

THE DEFINITIONS OF THE TERM “MATHEMATICS”

This research is mainly focused on the definitions, or descriptions, of the term “mathematics”, which are expressed as a single sentence, and have the form of a definition: definiens, followed by defining connective (usually “is”) and definiendum. A few exceptions which do not follow the described form are also in the list, only when the author considered that by their content, they are actually definitions of the term “mathematics”.

There are many different approaches to the classification of the definitions. We will follow the approach used by Wang, [37], where two main groups of definitions are differentiated,

- a) *dictionary definitions* – these are descriptive, they summarize the existing usage of the term and represent the widely accepted standard
- b) *working definitions* – these are prescriptive, they specify the proposed usage of the term and are usually proposed by a researcher or a research team.

Dictionary Definitions

As there exist special, mathematical dictionaries, we will distinguish definitions of the term “mathematics” that can be found in general dictionaries and those in mathematical dictionaries.

From General Dictionaries

Here we will list definitions that are intended for general addresses. They are taken from either a general dictionary or an encyclopedia. They report the general usage of the term.

- 1) *Oxford Learner's Dictionary*: The science of numbers and shapes. ([25])
- 2) *Collins Dictionary*: The study of numbers, quantities, or shapes. ([5])
- 3) *Macmillan Dictionary*: The study or use of numbers and shapes to calculate, represent, or describe things. ([18])
- 4) *The American Heritage Dictionary*: The study of measurement, properties and relationships of quantities and sets, using numbers and symbols. ([36])
- 5) *Cambridge Dictionary*: The study of numbers, shapes, and space using reason and usually a special system of symbols and rules for organizing them. ([4])
- 6) *Encyclopedia Britannica*: The science of structure, order, and relation that has evolved from elemental practices of counting, measuring, and describing the shapes of objects. ([12])
- 7) *Random House Kernerman Webster's College Dictionary*: The systematic treatment of magnitude, relationships between figures and forms, and relations between quantities expressed symbolically. ([21])
- 8) *Wikipedia, the Free Encyclopedia*: An area of knowledge that includes such topics as numbers (arithmetic and number theory), formulas and related structures (algebra), shapes and the spaces in which they are contained (geometry), and quantities and their changes (calculus and analysis). ([38])
- 9) *Meriam-Webster Dictionary*: The science of numbers and their operations, interrelations, combinations, generalizations, and abstractions and configurations of space and their structure, measurement, transformations, and generalizations. ([19])

From Mathematical Dictionaries

Mathematical dictionaries clarify mathematical terms to librarians, students, teachers, engineers, and scientists – all who need or use mathematics in their studies or work.

- 10) *The Penguin Dictionary of Mathematics*: The study of numbers, shapes, and other entities by logical means. ([24])
- 11) *Dictionary of Mathematics Terms by Barron's*: Mathematics is the orderly study of the structures and patterns of abstract entities. ([8])
- 12) *Collins Dictionary of Mathematics*: A group of related subjects, including algebra, geometry, trigonometry, and calculus, concerned with the study of number, quantity, shape, and space, and their inter-relationships, applications, generalizations and abstractions. ([3])
- 13) *Oxford Concise Dictionary of Mathematics*: The branch of human enquiry involving the study of numbers, quantities, data, shape, and space and their relationships, especially their generalizations and abstractions and their application to situations in the real world. ([9])

We note that the McGraw Hill Dictionary of Mathematics, 2nd edition, from 2003, does not define the term mathematics ([22]).

Working Definitions

We will consider two sub-groups here as well, non-scientific and scientific working definition.

Non-scientific working definitions

Non-scientific have philosophical, aesthetic, artistic, creative, humoristic, or social elements involved in its content. For easier surveying, we will arrange the definitions according to their content, being aware that some of the definitions might belong to more than one group.

Philosophical

- 14) Mathematics is the door and the key to the sciences (Roger Bacon, 1220-1292, philosopher, [33])

- 15) The essence of mathematics lies in its freedom (Georg Cantor, mathematician, 1845-1918, [31])
- 16) Mathematics is a thought moving in the sphere of complete abstraction from any particular instance of what it is talking about. (Alfred North Whitehead, mathematician and philosopher, 1861-1947, [39])
- 17) Mathematics is the science of the most complete abstractions to which the human mind can attain. (Alfred North Whitehead, mathematician and philosopher, 1861-1947, [Whitehead56])
- 18) Mathematics is the servant as well as the queen of the sciences, and she weaves a reach fabric of creative theory, which is often inspired by observations in the phenomenal world, but is also often inspired by creative insight that recognizes identical mathematical structures in dissimilar realization by stripping the realizations of their substance and concerning itself only with undefined objects and rules governing their relations. (Mina Rees, mathematician, 1902-1997, [27]).
- 19) Mathematics is a tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field. (Paul Dirac, 1902-1984, theoretical physicist, [32])
- 20) Mathematics is entirely independent of the physical world. At the same time... mathematical systems can often usefully serve as models for portions of reality, thus providing a basis for a theoretical analysis of relations observed in the phenomenal world. (Marshall Stone, 1903-1989 in [35]).
- 21) Mathematics is the abstract key which turns the lock of the physical universe (John Polkinghorne, 1930-2021, theoretical physicist, theologian)
- 22) Mathematics links the abstract world of mental concepts to the real world of physical things without being located completely in either (Ian Stewart, mathematician, 1945- , [6])
- 23) Formal mathematics is like spelling and grammar – a matter of the correct application of local rules. Meaningful mathematics is like journalism – it tells an interesting story. Unlike some journalism, the story has to be true. The best mathematics is like literature – it brings a story to life before your eyes and involves you in it, intellectually and emotionally. (Ian Stewart, mathematician, 1945- , [6])

Aesthetical

- 24) The mathematical sciences particularly exhibit order, symmetry and limitation; and these are the greatest forms of the beautiful. (Aristotle, philosopher and polymath, 4th century BC [29])
- 25) Mathematics reveals its secrets only to those who approach it with pure love, for its own beauty (Archimedes, mathematician, engineer, inventor, 3rd century BC)
- 26) Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. (Bertrand Russel, philosopher and mathematician, 1872-1970, [23])
- 27) Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality (Richard Courant, mathematician, 1888-1972, [26])

Artistic

- 28) Mathematics is, in its way, the poetry of logical ideas (Albert Einstein, 1879-1955, theoretical physicist, [1])
- 29) Mathematics is on the artistic side a creation of new rhythms, orders, designs, harmonies, and on the knowledge side, is a systematic study of various rhythms, orders, designs and harmonies. (William L. Schaaf, mathematics educator, 1898-1992, [7])
- 30) Mathematics is a form of poetry which transcends poetry in that it proclaims a truth, a form of reasoning which transcends reasoning in that it wants to bring about the truth it proclaims; a form of action, of ritual behavior, which does not find fulfillment in the act, but must proclaim and elaborate a poetic form of truth. (Bochner Salomon, mathematician, 1899-1982, [10])

Creative

- 31) The pure mathematician, like the musician, is a free creator of his world of ordered beauty. (Bertrand Russel, philosopher and mathematician, 1872-1970, [23])
- 32) Mathematics is the most beautiful and most powerful creation of the human spirit. (Stefan Banach, mathematician, 1892-1945, [34])

Social

- 33) Mathematics is a more powerful instrument of knowledge than any other that has been bequeathed to us by human agency. (Descartes, 1596-1650, philosopher and mathematician, [17])
- 34) Mathematics is the mirror of civilization and the emperor of whole academic world (Lancelot Thomas Hogben, zoologist and medical statistician, 1895-1975, [26])
- 35) Mathematics is a linguistic activity; its ultimate goal is preciseness of communication. (William L. Schaaf, mathematics educator, 1898-1992, [7])
- 36) Mathematics is a language which must be learned and the arsenal of techniques of mathematics must be mastered if we are to speak this language. Mathematics is both deductive and inductive, but like poetry, it needs persons who are creative and have a sense of the beautiful for its surest progress. (Mina Rees, mathematician, 1902-1997, [27]).
- 37) Mathematics can be seen as a human activity involving the solution of socially shared problem-situations, which refer to the real or social world or which are within the realm of mathematics itself. As a response or solution to these external and internal problems, mathematical objects (concepts, procedures, theories, etc.) progressively emerge and evolve. (Godino, researcher in Mathematics education, 1947-, and Batanero, researcher in Statistic and Didactics, [11])

Humoristic

- 38) Mathematics is a game played according to certain simple rules with meaningless marks on paper (David Hilbert, mathematician, 1862-1943, [15]).
- 39) Mathematics is the subject in which we do not know what we are talking about or whether what we say is true. (Bertrand Russel, philosopher 1872-1970, [23])
- 40) Mathematics is only the art of saying the same thing in different words. (Bertrand Russel, philosopher 1872-1970, [23])
- 41) Mathematics is the science which uses easy words for hard ideas. (Edward Kasner, 1878-1955, mathematician, and James R. Newman, mathematician 1907-1966, [7])

Scientific Working Definitions

The definitions in this group are given by scientists, mainly mathematicians and physicists, or educators.

- 42) Mathematics is a science devoted to the exploration of number, quantity, geometry, and in modern times also includes investigation into yet more abstract concepts of order, and into analogous types of purely logical relations. (Alfred North Whitehead, mathematician and philosopher, 1861-1947, [39])
- 43) Mathematics (in its widest significance) is the development of all types of formal, necessary, deductive reasoning (Alfred North Whitehead, philosopher and mathematician, 1861-1947, [10])
- 44) Mathematical ideas originate in empirics, although the genealogy is sometimes long and obscure. But, once they are conceived, the subject begins to live a peculiar life of its own and is better compared to a creative one, governed by almost entirely aesthetical motivations (John Von Neumann, mathematician, physicist, engineer, 1903-1957, [20])
- 45) Mathematics is nothing but a system of conclusions drawn from definitions and postulates that must be consistent but otherwise may be created by the free will of the mathematician. [6, 27]

- 46) Mathematics is the science of skilful operations with concepts and rules invented just for this purpose. (Wigner Eugene Paul, physicist, 1902-1995, [10])
- 47) Mathematics is the science of number and space. It starts from a group of self-evident truths and by infallible deduction arrives at incontestable conclusions... the facts of mathematics are absolute, unalterable and eternal truths (Stabler E. Russel, 1906-1971, mathematician, [10])
- 48) Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns—systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically (“pure mathematics”) or models of systems abstracted from real-world objects (“applied mathematics”). (Stephen Krulik, professor of mathematics education, 1933- , [28])
- 49) Mathematics is the science of order and mathematicians seek to identify instances of order and to formulate and understand concepts that enable us to perceive order in complicated situations (Shlomo Sternberg, mathematician, 1936- , [10])
- 50) Mathematics is the study of assumptions, its properties, and applications. (Dharmendra Kumar Yadav, current Assistant Prof. of Mathematics, [40])

AN ANALYSIS OF THE CLASSIFIED DEFINITIONS

The classification exposed in the last section, shows the essential content of the description of the word mathematics in the listed definitions.

The dictionary definitions are listed in order of growing complexity. In the general dictionaries' definitions, we can observe that mathematics is most often put in the general category of “science”, sometimes “study”, but it is also defined as an “area of knowledge”, and “systematic treatment”. The most frequent words used in the definiendum are: numbers, shapes, space, measurements, quantities, relations, symbols, structure, order, rules. We may say that the last two dictionary definitions (from Wikipedia and Merriam-Webster Dictionary) cover the biggest area of mathematics.

In the definitions from the mathematical dictionaries, we can observe that “abstraction”, “generalization” and “application” are added to the definiendum. Also, there are two new general categories added, “a group of related subjects” and the very interesting, “a branch of human enquiry”.

The working definitions are listed in chronological order, per group. The non-scientific working definitions abound with different personal perspectives of the creators, expressed in the sub-division: philosophical, artistic, aesthetical, creative, social, humoristic. In fact, this group of definitions depicts the long-lasting tight relationship of 20 centuries between mathematics and philosophy. Since “The highest form of pure thought is in mathematics.” (Plato, philosopher, 5th century BC) and “A mathematician who is not somewhat of a poet, will never be a perfect mathematician” (Karl Weierstrass, mathematician, 1815-1897, [7]). The phrases used in this group of definitions which we would like to distinguish here as a prominent part of the definiendum to attribute the word “mathematics”, are: pure love, supreme beauty, ordered beauty, greatest forms of the beautiful, aesthetic perfection, intellectual and emotional involvement, creative theory, the poetry of logical ideas, a poetic form of truth, creation of the human spirit, phenomenal world, key to the universe, emperor of the whole academic world.

Scientific working definitions are mainly created by mathematicians, physicists, and mathematical educators. They depend on the personal viewpoint to a large extent. We can observe the words: deduction, deductive reasoning, analogy, assumption, postulates, axioms, exploration, experimentation, theory, abstraction, empirics, and application, in this group of definitions. In general, there are two different standpoints, one in which there is an emphasis on abstraction, and the other, that emphasizes the application of mathematics. Out of the nine listed definitions in this group, only three of them (numbered 44, 48, and 50) emphasize the application of mathematics.

The scientific working definition will be considered good enough if it is similar to the dictionary definition, it is exact and simple, covers all common usage of the term, and gives the term the proper identity. However, due to the complexity of the term “mathematics”, none of the listed definitions satisfies these properties.

CONCLUSION

There is no single definition of the term “mathematics”, that the larger group of mathematical learners and practitioners will be satisfied with. However, there are plenty of descriptions, meaningful, luminous, inciteful, insightful, penetrating, and profound. We were curious to find out the variety of the definitions throughout the literature and eager to collect both formal and nonformal definitions.

A general division of the collected definitions is made, on dictionary and working definitions and further subdivisions on general and mathematical dictionaries' definitions, for the former group, and non-scientific and scientific, for the latter group. There is a total of 50 definitions, out of which 13 are dictionary definitions (9 from general dictionaries and 4 from mathematical dictionaries), and 37 are working definitions (9 are scientific working definitions and among the 28 non-scientific definitions, 10 are philosophical, 5 are social, 4 - aesthetical, 3 - artistic, and 2 – creative). In most of the definitions, the word “mathematics” is put in the general category of “study”, “science” or “area”, and the attributes mostly used to describe the word are numbers, shapes, space, measurements, quantities, relations, symbols, structure, order, rules, but also: love, beauty, poetry, creation, key, perfection and so on...

Different perspectives give rise to contextually different definitions, adjusted to the envisioned usage of the word “mathematics”. We note the dependence on personal (subjective or mental) dimension, the institutional (objective, contextual) dimension, but also the dependence on the period in history, tradition, stage of the mathematical development of the creator, and the target audience.

Many people do not mind that the term “mathematics” is not defined. They just understand it to the extent to which they use mathematics. It is unrealistic to expect a commonly accepted definition, but it would be of great benefit if a consensus is reached. Following the iterative method, or method of successive improvements (starting with a preliminary definition, identifying its shortages, and improving it), hopefully, the term “mathematics” will obtain a satisfactory definition. However, a fair amount of brilliance will be required to construct an effective definition. According to Jourdain, “there is a great tendency the question as to what mathematics is could be answered if we knew all the facts of the development of our mathematical knowledge” [14].

Including the listed definitions or their part, in at least one hour in a mathematical or philosophy class will help students get into aspects of

mathematics they were most likely have not been aware of, and enrich their mathematical experience. It will also help teachers to actively communicate mathematics to students.

We would like to emphasize that the positions in this review of the definitions of the term “mathematics”, are stated from a perspective of a mathematician. Further analysis might be done from a historical, philosophical, linguistic, semantic, pragmatic, or another perspective.

REFERENCES

- [1] Albert Einstein Quotes. (n.d.). BrainyQuote.com. Retrieved May 28, 2022, from BrainyQuote.com Web site:
https://www.brainyquote.com/quotes/albert_einstein_106697
- [2] Alcock, L. and Simpson, A. (2017), Interactions between defining, explaining and classifying: the case of increasing and decreasing sequences, *Educational studies in mathematics*, 94 (1), pp. 5-19.
- [3] Borowski E. J., Borwein J. M. (2002) *Collins Dictionary of Mathematics*, 2nd edition, Collins, London.
- [4] Cambridge Dictionary: Mathematics,
<https://dictionary.cambridge.org/dictionary/english/mathematics>.
Accessed 25 May 2022.
- [5] Collins Dictionary: Mathematics,
<https://www.collinsdictionary.com/dictionary/english/mathematics>.
Accessed 25 May 2022.
- [6] Courant R., Robbins H., revised by Stewart I. (1996), *What is Mathematics?*, second edition, Oxford University Press.
- [7] Curcic, B. (1976), Mathematical Quotations for All Occasions, *The Mathematics Teacher*, 69(1), 40–44.
<http://www.jstor.org/stable/27960358>
- [8] Downing D. (2009), *Dictionary of Mathematics Terms*, Third Edition, Barron’s Professional Guides.
- [9] Earl R., Nicholson J. (2021) *Oxford Concise Dictionary of Mathematics*, Oxford University Press.
- [10] Gaither C. C., Cavazos-Gaither E. A. (2012), *Gaither’s Dictionary of Scientific Quotations*, 2nd edition, Springer
- [11] Godino, Juan & Batanero, Carmen. (1998), Clarifying the Meaning of Mathematical Objects as a Priority Area for Research in Mathematics Education. 10.1007/978-94-011-5190-0_11.

- [12] Gray, Jeremy John , Fraser, Craig G. , Berggren, John L. , Folkerts, Menso and Knorr, Wilbur R.. "mathematics". *Encyclopedia Britannica*, 9 Nov. 2020, <https://www.britannica.com/science/mathematics>. Accessed 27 May 2022.
- [13] Hanson, S. O. How to define: a tutorial. *Principles: Journal of Philosophy (UFRN)*, [S. l.], v. 13, no. 19-20, p. 05–30, 2010. Available at: <https://periodicos.ufrn.br/principios/article/view/508>. Accessed on: 17 May. 2022
- [14] Jourdain E. B. (1956), *The Nature of Mathematics, The World of Mathematics*, Vol. 1, ed. James R. Newman, London, Georg Allen and Unwin LTD.
- [15] Kramer E. E. (1983), *The nature and Growth of Modern Mathematics*, Princeton University Press, Princeton New Jersey.
- [16] Lackmann M., Schleicher D., editors (2011), *An invitation to Mathematics*, Springer-Verlag Berlin Heidelberg (Preface: What is Mathematics? by G unter M. Ziegler)
- [17] Lagemaat, R. (2015) *The Theory of knowledge for the IB diploma*, second edition, Cambridge University Press, 2015
- [18] Macmillan Dictionary: Mathematics, <https://www.macmillandictionary.com/dictionary/british/mathematics>. Accessed 25 May 2022.
- [19] Merriam-Webster Dictionary, since 1828: Mathematics <https://www.merriam-webster.com/dictionary/mathematics>. Accessed 25 May 2022
- [20] Neumann J. V. (1947), *The Mathematician, Works of the Mind*, Vol. 1, no1, University of Chicago Press, Chicago, 180-196.
- [21] Mathematics. (n.d.) Random House Kernerman Webster's College Dictionary. (2010). Retrieved May 28 2022 from <https://www.thefreedictionary.com/Mathmatics>.
- [22] McGraw-Hill Dictionary of Mathematics, 2nd edition (2003), McGraw-Hill Professional
- [23] McMaster University, Quotations from Bertrand Russel, <https://www.mcmaster.ca/russdocs/brquotes.htm>, Accessed: 18 May 2022.
- [24] Nelson D. (2008), *The Penguin Dictionary of Mathematics: Fourth Edition*, Penguin Books.
- [25] Oxford Learner's Dictionary: Mathematics, <https://www.oxfordlearnersdictionaries.com/definition/english/mathematics?q=mathematics>. Accessed 25 May 2022.

- [26] Rani T. S. (2008), Teaching of Mathematics: Modern Methods, A.P.H. Publishing Corporation, New Delhi, India.
- [27] Rees, M. (1962). The nature of mathematics, The Mathematics Teacher, 55 (6), 434–440, <http://www.istor.org/stable/27956649>.
- [28] Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), Handbook for Research on Mathematics Teaching and Learning, pp. 334-370, New York: MacMillan.
- [29] St. Andrews University, Mac Tutor Quotations from Aristotle, <https://mathshistory.st-andrews.ac.uk/Biographies/Aristotle/quotations/> Accessed 18 May 2022
- [30] St. Andrews University, Mac Tutor Quotations from Galileo Galilei, <https://mathshistory.st-andrews.ac.uk/Biographies/Galileo/quotations/#:~:text=%5BThe%20universe%5D%20cannot%20be%20read,to%20comprehend%20a%20single%20word>, Accessed: 19 May 2022.
- [31] St. Andrews University, Mac Tutor Quotations from Georg Cantor, <https://mathshistory.st-andrews.ac.uk/Biographies/Cantor/quotations/> Accessed: 19 May 2022
- [32] St. Andrews University, Mac Tutor Quotations from Paul Dirac, <https://mathshistory.st-andrews.ac.uk/Biographies/Dirac/quotations/> Accessed: 20 May 2022
- [33] St. Andrews University, Mac Tutor Quotations from Roger Bacon, <https://mathshistory.st-andrews.ac.uk/Biographies/Bacon/quotations/> Accessed: 20 May 2022
- [34] Stefan Banach Quotes. (n.d.). BrainyQuote.com. Retrieved May 28, 2022, from BrainyQuote.com, https://www.brainyquote.com/quotes/stefan_banach_311403.
- [35] Stone M. (1961), The Revolution in Mathematics, Liberal education, XLVII, p. 307.
- [36] The American Heritage Dictionary: Mathematics, <https://ahdictionary.com/word/search.html?q=mathematics>. Accessed 25 May 2022.
- [37] Wang, P. (2019), On Defining Artificial Intelligence. Journal of Artificial General Intelligence, 10(2) 1-37. <https://doi.org/10.2478/jagi-2019-0002>.
- [38] Wikipedia, the free Encyclopedia: Mathematics, <https://en.wikipedia.org/wiki/Mathematics>. Accessed 25 May 2022

- [39] Whitehead A. N. (1956) Mathematics as an Element in the History of Thought, The World of Mathematics, Vol. 1, ed. James R. Newman, London, Georg Allen and Unwin LTD.
- [40] Yadav, K. D. (2017). Exact Definition of Mathematics, International Research Journal of Mathematics, Engineering and IT, Vol. 4, Issue 1, 34-42.