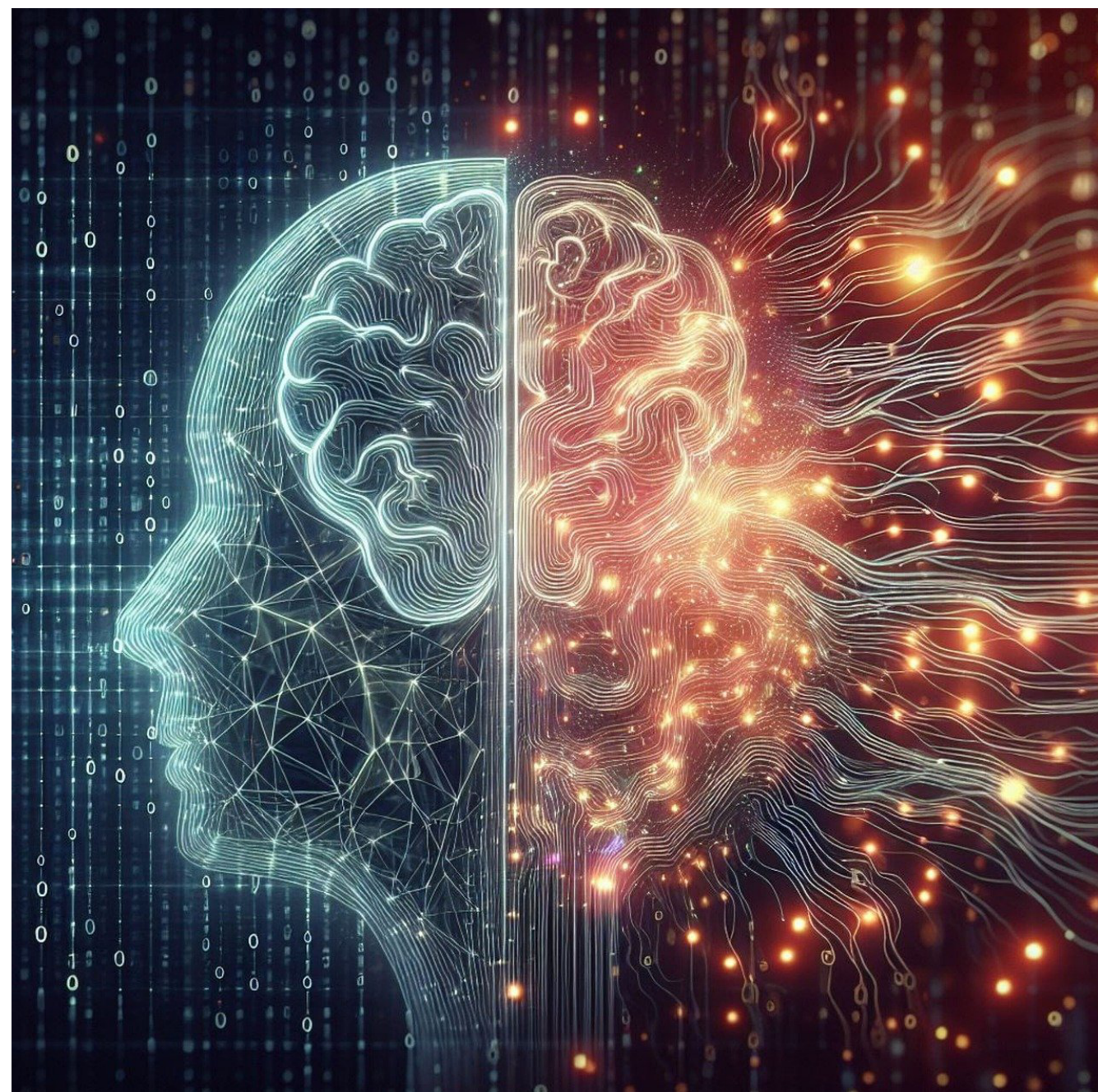


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A MATHEMATICAL CONCEPT IN LENS-BASED MEDIA: REVISITING THE GOLDEN RATIO

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Abstract

Mathematical essential concepts tend to be firmly rooted in traditional teaching methods across all levels of education. High school mathematics teachers are challenged to respond to the expectations and requirements of contemporary learners, commonly characterized as „Z-generation”. Efforts must be made to make the mathematical lessons more engaging and connected to the real world, but of course, not at the cost of the mathematical correctness. In this paper we are revisiting the Golden Ratio, a number which enables making the abstract concept of irrational numbers tangible and lifeful. We present the application of the golden ratio in lens-based media as one of the strongest mathematical principles that can be used to create aesthetically pleasing images. In photography and cinematography, composition is the art of framing the scene and arranging the elements within the frame. It is the most critical and challenging task every artist faces. Creating a dynamic and well-balanced composition helps artists express themselves and capture the audience's attention. The Golden Ratio provides a basis for some of the most fundamental compositional techniques that enable artists to harmoniously divide the picture space and arrange the compositional elements in relation to one another and the whole.

Keywords: mathematics, art, Z-generation, irrational numbers, golden ratio, lens-based media, compositional techniques

Introduction

Although „For more than two thousand years some familiarity with mathematics has been regarded as an indispensable part of the intellectual equipment of every cultured person” [6], nowadays mathematical classes are perceived as lifeless and chore by most of the students. We will here focus our attention to the Z-generation of students (born between 1995 and 2010), more specifically contemporary high school students. These students have grown up with technology, internet, social networks, and games, often looking at more than one screen at the same time or doing other form of multitasking. As a result, they tend to exhibit a lack of focus and impatience in general. They have bigger expectation from teachers, in their teaching style and communication, and can't easily interact

if their preferences for digital literacy, experiential learning, interactivity, visual forms, and immediacy are not satisfied. [24], [27]

While for the newer generations of students it is becoming increasingly challenging to maintain precision, concentration, and the ability to memorize things for an extended period, mathematics, as a coherent, harmonious, structured and systematized knowledge, requires attention, focus, patience, and persistency. These counter positions certainly make mathematics in a grave danger to be undesirable subject. Moreover, mathematics is one of the courses that is most rigid and resistant to changes in teaching style. Actually, it is a course mostly connected to the traditional ways of teaching: a teacher must explain the basic mathematical concepts by writing on board (either traditional or digital board). Then students must patiently follow the teachers' writings, symbol by symbol, to be able to comprehensively understand the mathematical idea. The knowledge has to be absorbed continuously and gradually, from the subsequent presentation of facts, diving deeper and deeper into the content. Additionally, practicing and doing homework is a meaningful part of the process of learning mathematics. Traditional tests that necessitate handwritten solutions can't be easily replaced by multiple-choice, matching items, or true-false tests. The standard strategies and methods in teaching mathematics seem so inadequate to Z-generation and moreover, to Alpha generation of students (born after 2010).

Bringing new and innovative mathematical lessons in the classroom to actively involve students, while maintaining mathematical rigor, is not an easy task. Majority of today's high-school teachers belong to X and Y generations, born between 1965 and 1979 for the former, and 1980-1995, for the later. (The time limitations of X, Y, Z and alpha generations differ in the literature; thus, the given yearly limitations should be taken as approximative. More on characteristics on generations can be read in [Csobanka 2016] or [Crindle 2009]). They are expected to teach in a way they were neither taught nor prepared in their graduate studies. However, teaching style need changes as students from each generation "have right to have appropriate training to expand and strengthen their skills". [4]

Krupa in 2022 states that „By engaging students in unconventional activities, educators can connect mathematics to real-world applications that make the lessons meaningful and relevant for students.” [16] Similar conclusions were brought in research conducted in USA between July and December 2022. [12] We highlight two of them: „The fundamental priority among parents, teachers, and the public at large when it comes to math education is making instruction more engaging and real-world relevant for today's learners”, and „Math is identified as the subject most in need of updating and improvement in terms of the way it is taught.”

This paper aims to provide an illustration of how a mathematical concept can be applied in the realm of art composition, thus rendering the mathematical idea applicable and relevant to real-world scenarios. Namely, we will try to make the abstract concept of irrational numbers tangible by involving vivid examples of application of Golden Ratio $\Phi = \frac{1+\sqrt{5}}{2}$ in composition in lens-based media.

Composition and balance play a crucial role in photography and filmmaking, shaping the way artists frame their scenes and arrange the elements within the frame. Composition is the process of selecting, arranging, and emphasizing the components of an image to support the intended message or effect. It involves considering diverse objects, lines, shapes, patterns, colors, and negative space. Artists can express themselves and capture the audience's attention by creating a dynamic and well-balanced composition.

The Golden Ratio, also known as the Golden Section or Divine Proportion, is recognized as a mathematical principle as well, widely used in art and design to create aesthetically pleasing compositions. Throughout history, renowned artists, architects, and designers have employed the Golden Ratio as a guiding principle in their works. Structures like the Egyptian pyramids and the Parthenon on Acropolis are believed to incorporate proportions closely aligned with the Golden Ratio. Artists like Leonardo da Vinci and Le Corbusier intentionally used the Golden Ratio to achieve harmonious and balanced compositions in their artworks and architectural designs.

In lens-based media, such as photography and cinematography, artists face the challenge of working within the spatial confines of the viewfinder frame. While adhering to strict geometric principles may not always be possible, photographers and cinematographers can use compositional techniques derived from the Golden Ratio principle to guide the placement of subjects, lines, and elements within the frame. Some of these techniques include Golden Points (Phi Grid), which involve positioning key elements at strategic locations determined by the Golden Ratio, and the Golden Spiral, a logarithmic spiral that can guide the viewer's eye through the frame. Another widely used technique is the Rule of Thirds, which suggests dividing the frame into thirds and placing areas of interest along the lines or at their intersections.

While these techniques provide valuable guidelines, it's important to view them as tools that can be adapted or broken to suit the artist's vision. Achieving a well-balanced composition also requires considering the relationships between elements, the concept of visual weight, and the overall interaction of components within the frame. By balancing adherence to guidelines with an intuitive approach to composition, artists can create captivating and harmonious visual narratives.

Irrational numbers: Conceptions AND misconceptions

During their mathematical education in primary and secondary school, the students are introduced to a progression of number sets, starting with natural numbers, followed by non-negative rational numbers, integers, rational numbers and ending with irrational and real numbers. Real numbers represent all the quantities that can be found in the real world, which is a reasonable and acceptable rationale of why high-school students should well understand these numbers. And comprehensive understanding of the concept of real numbers will not be possible without at least basic understanding of irrational numbers. Some scientists consider the invention and understanding of irrational numbers as one of the greatest discoveries in science, e.g. [1]

Irrational numbers are defined either as non-repeating infinite decimals or non-fractions. They can be also defined as “the numbers that are not rational”, when the definition of rational numbers is clear. Before synthetizing the general definition, many textbooks prefer to introduce irrational numbers by addressing the relationship between the diagonal and the side measure of the square. [21] Or similarly, the relationship between hypotenuse and cathetes. In such a way the students are acquainted by $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ or other square roots which are irrational numbers. The irrational number π is typically introduced as a quotient between the circumference and diameter of a circle. Later the irrational Golden Ratio ϕ is introduced (the notation ϕ for the Golden Ratio was suggested in the beginning of the 20th century, after the ancient Greek sculptor Phidias ([Huntley1970])),

$$\phi = \frac{1 + \sqrt{5}}{2}$$

There are two mostly used approaches in introducing the Golden Ratio. The first, simpler one, is with the line segment AB of length l. This segment is divided with the point M on two subsegments: AM of length denoted by x, and MB of length l-x. The point M is chosen such that

$$\frac{\overline{AB}}{\overline{AM}} = \frac{\overline{AM}}{\overline{MB}}, \text{ i.e. } \frac{1}{x} = \frac{x}{l-x}.$$

The last equality may be written in its equivalent form,

$$x^2 = l^2 - lx$$

which after division by x^2 , and using the notation $\phi = \frac{1}{x}$, gets the form of square equation in the unknown ϕ ,

$$\phi^2 - \phi - 1 = 0.$$

Its positive solution is exactly the Golden Ratio,

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.61803398875 \dots$$

and the negative solution is not considered, since $\phi = \frac{1}{x}$ must be positive, as a quotient of lengths.

The other way of introducing the golden number is through Fibonacci sequence. Fibonacci sequence is defined recursively, such that its first two elements are 1 and 1 and every next element is the sum of the last two elements. In mathematical symbols,

$$f_1 = 1, f_2 = 1, f_{n+2} = f_{n+1} + f_n, \quad n = 1, 2, 3, \dots$$

The elements of this sequence are the following

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots$$

It can be proved that the sequence

$$\left\{ \frac{f_{n+1}}{f_n} \right\}_{n=1}^{\infty} : 1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \dots \quad (*)$$

converges and its limit is exactly the golden ratio,

$$\lim_{n \rightarrow \infty} \frac{f_{n+1}}{f_n} = \phi = \frac{1 + \sqrt{5}}{2}.$$

In some high-schools, or first year of an undergraduate study, the introduction of Golden Ratio continues with proving that Golden Ratio is indeed an irrational number. Based on the fact that $\sqrt{5}$ is an irrational number and the sum of a rational and irrational number is also an irrational number, one can conclude that ϕ is also an irrational number.

The preceding paragraphs aim to illustrate the highly abstract nature of the mathematical approach when introducing irrational numbers. Having difficulties to comprehend this theoretical portion, a sense of aversion typically emerges among majority of students. Such circumstances necessarily yield to deficiencies in this matter.

In a research study performed in 2018 [17] on students' conceptions of irrational numbers the following findings were noted. „In spite of the fact that around 80% of the students claimed that they had learned about irrational numbers, only a small percentage of students (19%) showed awareness of the existence of irrational numbers as non-repeating infinite decimals. The same percentage of

students (20%) showed awareness of the existence of irrational numbers on the real number line.” In another research from 2022 the subsequent conclusions were derived „for most undergraduate students irrational number means indeterminate due to infinite decimal digits (that’s why many students considered that the fraction $\frac{1}{3}$ is an irrational number), in the sense that irrationality is equivalent to ignorance, while in terms of placement of numbers, our results made clear that the dominant strategy of the students was to convert the given numbers into decimals, round them and then place them on the number line, ignoring the geometric methods taught”. [22]

Not only for high-school students, misunderstandings and misconceptions are noted even at teachers’ conceptual understanding of irrational numbers.

Taking into account that basic mathematical concepts are primarily comprehended in high school, and very rarely later, this finding is to be anticipated. Fischbein [9] surveyed the knowledge of high school students and prospective teachers regarding the irrational numbers and concludes that „most of the high school students and many preservice teachers are not able to define correctly the concepts of rational, irrational and real numbers.” Guler [13] was analyzing primary school mathematics teachers’ conceptual understanding of irrational numbers and concludes that „According to the interviews, mathematics teachers were not able to correctly define rational and irrational numbers” and also „Teachers had difficulty placing rational and irrational numbers in sequence.” Similarly, Zazikis and Sirotic [23] investigated the understanding of irrational numbers of a group of prospective secondary school mathematics teachers and found that „understanding in this area is relatively poor” and they „call for reconsideration of instructional and pedagogical approaches to irrational numbers.”

Approximation of irrational numbers with rational numbers is known concept in mathematics. [20], [21] Wagner in his book „Irrational Numbers: A constructive approach at Elementary and High School” considers the possibility of better understanding irrational numbers through their approximations with rational numbers. Having these approximations in mind, students will enhance their ability to distinguish between rational and irrational numbers, which is one of the identified misconceptions, e.g. [9] Moreover, this link between rational approximations of irrational numbers is good to be known in the current digital era, since computers, due to the finite memory space, recognize irrational numbers only as their rational approximations. Particularly, when Golden Ratio is utilized in art and design, the approximations 1.618 (expressed as 1:1.618 or 1:0.618:1) and 2 (expressed as 2:1 or 1:1:1) are often used. These approximations can be observed in the elements of the sequence given in this section. Assuming that the teachers possess a solid knowledge of the concept of irrational numbers and their rational approximations, and can foster understanding in the classroom, linking

irrational numbers to real-world applications could potentially enhance student engagement and cultivate interest in this significant mathematical topic.

Compositional techniques based on the Golden Ratio. Composition and balance

What is composition, and why is it essential in photography and filmmaking?

In photography and filmmaking, the first and probably most obvious component is that there is a frame, and the uniqueness of a given artwork comes from what the artist puts in the frame, where he or she cuts the 360 degrees world around. Cropping people, for example, is perfectly acceptable, but the critical question is where do we chop them off, and where do we position the points of interest, like the eyes? Well, this is where composition comes in. Composition is the art of framing the scene and arranging the elements within the frame. Richard Garvey-Williams, in his book titled „Mastering Composition: The Definitive Guide for Photographers“, defines composition as a process of selecting, arranging, and emphasizing the component parts of the image to support the intended message or effect. [11] These components, or compositional elements, include diverse objects, contrasting areas of tone and color, lines, shapes, patterns, negative space, etc.

Creating a dynamic and well-balanced composition helps artists express themselves and capture the audience's attention. It is probably the most critical and challenging task every artist faces when creating an artwork. As human beings, we have an innate sense of balance and project that sense onto the artwork. That is why specific patterns and alignment methods of the image's parts are pleasing, and a lack of balance could irritate the audience and impair communication. Artists and scientists have long sought to understand what makes a successful and balanced composition, and a series of composition rules have been established over the years. One of the strongest mathematical principles that can be used to create aesthetically pleasing images is the „Golden Mean“, also known as „Golden Ratio“, „Golden Section“, or „Divine Section/Proportion“. The Golden Ratio principle provides guidelines on how to harmoniously divide the picture space and arrange the compositional elements in relation to one another and the whole.

However, before we discuss some of the compositional techniques based on the mathematical principle of the Golden Ratio, it is important to acknowledge that the word rules probably is not the most appropriate when discussing art composition. They should be treated as guidelines because they are designed to be adapted or broken. Photographs or films would be boring if everyone composed every shot the same way. The artist aims to look for imaginative ways to adjust or take advantage of these guidelines.

We should also mention that besides these rules that help to divide the image space harmoniously, there are some other considerations to take into account when composing your image. The good and well-balanced composition sometimes is not only a result of positioning one element according to these rules but also of the relationship between the diverse elements. Gestalt psychology, for example, explains composition as having properties that cannot be derived from a simple summation of its parts. The conviction developed that the whole is more than the sum of its parts. A simple addition of isolated parts cannot perceive this whole. As Richard Garvey-Williams says, it's crucial to consider how all the significant and minor components we can see through the viewfinder interact and function as a whole. [11]

Another consideration worth mentioning is that positioning the elements also depend on a concept known as „visual weight“. Visual weight is the visual dominance of the element, i.e., how strongly the element attracts the audience's attention. This concept is particularly important when trying to create asymmetrical balance. Amy E. Arnston in „Graphic Design Basics“ claims that asymmetrical compositions evoke a great sense of movement and change, and balance is not a state of rest but equal tension. She explains that balance is achieved by two forces of equal strength that pull in opposite directions or by multiple forces pulling in different directions whose strengths offset one another. We can think of visual balance as a multiple rope pull where, for the moment, all teams exert the same power on the rope. [2] Some characteristics that determine the visual weight are size, value, color, isolation, etc.

To summarize, the key to a successful composition is finding a balance between adhering to guidelines, i.e., positioning the elements according to the guidelines, identifying a sense of harmony by evaluating the relationships among the different elements within the picture space, as well as employing an intuitive approach to composition.

Golden Ratio in art history

As previously mentioned, the principle of the Golden Ratio is one of the strongest and most fundamental compositional rules used in visual arts and design. It helps and is a good starting point for creating a unified, well-balanced composition. Throughout history, numerous artists and architects have employed the Golden Ratio as a guiding principle in their works, often utilizing the golden rectangle. This mathematical proportion has found its way into celebrated masterpieces across the globe. Notable structures like the Egyptian pyramids and the Parthenon on Acropolis suggest that numerous architectural proportions closely align with the Golden Ratio. According to the Greek historian Herodotus, the pyramid's construction was such that the area of each face equaled the area of a square, with sides equal to the pyramid's height. [14] This implies that the relationship

between the pyramid's faces and its height follows the divine proportion. There is no definitive proof regarding whether the architects of the Great Pyramid intentionally incorporated this relationship. Extensive literature and research support and refute this theory. Nonetheless, as Hemenway [14] explains, it remains an intriguing speculation, particularly considering the visual elegance of the pyramid. It is highly likely that some form of ratios and proportional considerations were considered during its design and construction.

Doczi [8], in his book „The Power of Limits: Proportional Harmonies in Nature, Art, and Architecture”, explains that the Parthenon's main facade fits into a single reclining golden rectangle. Also, the top of the column capitals approximates the golden section point of the total height. The architects understood the Golden Ratio and intentionally incorporated it into their designs. Alternatively, it is plausible that the architects relied on their inherent sense of aesthetic balance, resulting in proportions that naturally approximated the Golden Ratio.

An iconic illustration of this can be seen in Leonardo da Vinci's renowned drawing, the Vitruvian Man, where the human figure is depicted with a ratio of 1.62, symbolizing an ideal of perfection. Clearly, Leonardo da Vinci was one of the artists who loved the Golden Ratio, also known as the divine proportion during the Renaissance era, and extensively employed it in his artworks. For example, in „The Last Supper”, all the significant dimensions of the room and the table were derived from the Golden Section. This demonstrates Da Vinci's deliberate utilization of this mathematical principle to achieve harmonious and balanced compositions. [14]

A more recent example could be found in the work of the Swiss-French architect, designer, painter, urban planner, writer, and one of the pioneers of what is now regarded as modern architecture – Le Corbusier. Hemenway [14] explains that Le Corbusier devised a proportional system known as Modulor to eliminate the need for converting measurements between different systems such as feet, inches, and the metric system. Initially, he utilized a small set of measurements derived from human body proportions on his construction sites. Expanding upon this concept, Le Corbusier incorporated the golden section and the Fibonacci series of numbers to refine the Modulor system. Consequently, his buildings showcase the application of these proportions, demonstrating the influence of the Modulor system in his architectural designs.

Golden Ratio in lens-based media and some of the derived compositional techniques

Photographers and cinematographers work within the spatial confines of the viewfinder frame, where the elements present automatically establish divisions within the picture frame. While countless divisions are possible, the most captivating ones are those that possess a noticeable relationship with one

another. Division, at its core, revolves around proportion and, as we mentioned earlier, is a subject that has captivated artists throughout different epochs in history. The Golden Ratio is one of the best-known harmonious divisions. As Brown [3] says, classical philosophy expressed the idea that mathematics was the controlling force of the universe and that it was expressed in visual forces in the golden mean.

When it comes to lens-based media, the Golden Ratio is somewhat complicated and challenging to use when trying to compose your shot through the viewfinder. Artists working with a camera do not have freedom like the painters, for example, to place elements exactly where they want. Much of what they do requires compromises and balancing the guidelines with an intuitive approach to composition. As Freeman [10] explains, when dividing a frame into areas, the most valuable method is to train your eye by developing a familiarity with the subtleties of harmony in various proportions. By cultivating this understanding, the intuitive composition becomes increasingly refined. We may choose to overlook the strict principles of geometry, but we cannot disregard the inherent satisfaction that arises from these proportions.

The following section covers some of the most popular compositional techniques based on the Golden Ratio that offer photographers and cinematographers powerful guides for placing subjects, lines, and elements within the frame, enhancing the overall aesthetic appeal and drawing the viewer's attention to specific points of interest.

Golden Points (Phi Grid)

Golden Points, derived from the Golden Ratio, are strategic locations within a composition that hold significant visual impact. The Phi Grid divides the frame into nine sections and applies the Golden Ratio, 1:0.618:1 ratio, to determine the width of the sections. Placing the key elements, such as the main subject or important elements of interest, at these Golden Points can enhance the overall balance and harmony of the composition.

Figure 1. Bakreski Y. (2014). Road Series



In the example provided, the little girl holds the main focus, and therefore, it is positioned along one of the vertical divisions, also at two of the Golden Points. The most exciting part of the scene is happening around the small, central rectangle delineated by the Phi Grid. It is important to note that an image can also incorporate multiple elements or lines arranged in accordance with these principles.

Cinematographers often compose shots of the human face in alignment with the Phi Grid. Within the structure of the face, the eyes are typically positioned at the Golden Ratio between the hairline and the chin. Similarly, one can envision the screen as a face looking back at the audience while the audience gazes at it. In this analogy, the top of the frame corresponds to the hairline, and the bottom fits to the chin.

Figure 2. By placing the eyes of the subject within the frame in a manner consistent with their positioning in a human face, a sense of naturalness and aesthetic balance is achieved.



The Golden Spiral

The Golden Spiral is a logarithmic spiral that expands at a rate of ϕ , which means that each quarter turn of the spiral has a constant increase in size that approximates the Golden Ratio. This spiral pattern is found in nature and is often considered aesthetically pleasing. It is also used in art to create visually appealing compositions.

Figure 3. One famous example of a Golden Spiral in the art can be found in the woodblock print „The Great Wave off Kanagawa” by the Japanese artist Hokusai

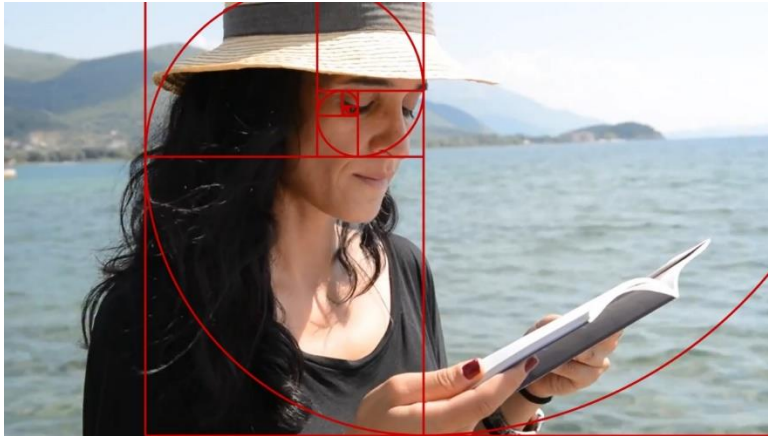


The artwork depicts a powerful ocean wave and three boats moving through the storm, with Mount Fuji in the background. The wave is depicted in a spiral shape, which can be seen by tracing the path of the foam at the crest of the wave. The spiral follows the logarithmic expansion of the Golden Ratio, creating a sense of motion and energy in the composition. The use of the Golden Spiral in this painting is considered intentional, as Hokusai was known to incorporate mathematical principles into his artwork. The Great Wave off Kanagawa is a prime example of how the Golden Spiral can create a sense of harmony and balance in art.

The Golden Spiral can be a valuable compositional tool in lens-based media. Artists working with the camera can create visually engaging images that guide the viewer's eye through the frame by incorporating the spiral's logarithmic growth pattern. One common application uses the Golden Spiral to position key elements within the picture, such as leading lines, curves, or focal points. This helps create a sense of balance, harmony, and aesthetic appeal. In filmmaking, the Golden Spiral could be used to emphasize movement. Filmmakers may also incorporate spiral motifs in set design, props, or costume choices to further reinforce the presence of the Golden Spiral. Additionally, post-processing techniques can be employed to subtly overlay the Golden Spiral grid on an image, allowing for precise alignment and further emphasizing the spiral's influence. Overall, the

Golden Spiral offers a powerful tool to enhance the visual impact of the images and create captivating compositions.

Figure 4. Bakreski Y. (2014). Small Nation Poet. (A Film Still)



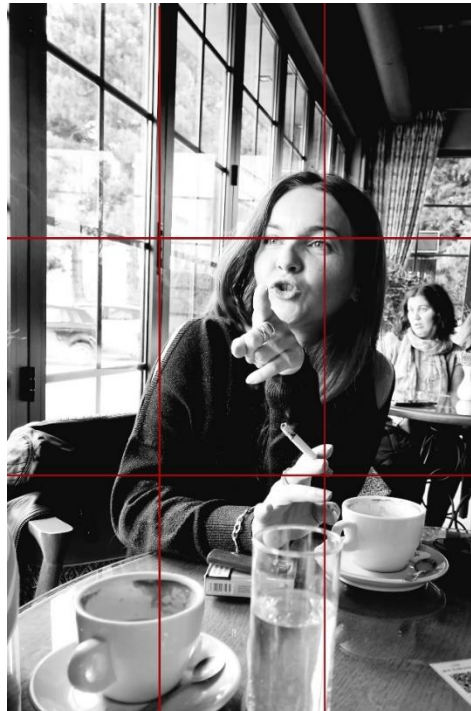
Using the Golden Spiral in film composition adds a layer of visual sophistication, enhances the overall viewing experience, and achieves a level of artistic accuracy that heightens the impact of their storytelling on the audience.

Regarding lens-based media, achieving a well-composed photograph using the Golden Spiral would likely require a clear visualization of the spiral and meticulous planning to effectively align the elements within the frame. However, as Richard Garvey-Williams writes in his book on composition, it is worthwhile to observe naturally occurring spirals in the surroundings that fit this design, as they can enhance the visual appeal of the shot. [11]

The Rule of Thirds

The Rule of Thirds is one of the fundamental compositional techniques. It was first recorded in 1797 by John Thomas Smith in his book „Remarks on Rural Scenery“. The rule suggests dividing the screen into three equal columns and rows with four lines. Your areas of interest should fall near one of the lines, ideally at the intersection of two of these lines. These lines are good positions for placement of any strong lines in the scene, like the horizon, etc. John Thomas Smith describes it in the following way: „For example, in a design of landscape, to determine the sky at about two-thirds; or else at about one-third, so that the material objects might occupy the other two: Again, two-thirds of one element, (as of water) to one-third of another element (as of land); and then both together to make but one-third of the picture, of which the two other thirds should go for the sky and aerial perspectives.“ [25]

Figure 5. Bakreski Y. (2022). At the Café



There is a lot in this photograph that follows the Rule of Thirds. In a portrait like this, the eyes are the primary area of interest, so they are placed at the intersection of the top horizontal line and the right vertical line. Moreover, the tips of the fingers holding the cigarette also touch the right vertical line.

Smith continues to explain that the ratio of about two-thirds to one-third, or two to one, is much better and more harmonizing than any other proportion. He explains that his invention could be applied to any specific case, such as light, shade, form, or color. [25]

While exploring the Rule of Thirds, Edwin Westhoff discovered another technique called The Diagonal Method. [11] Through observation, he noticed that artists were naturally inclined to position essential details along the bisection lines formed by dividing the 90-degree corners of their work into two equal 45-degree angles. This discovery shed light on an instinctive tendency among artists to utilize these lines as a means of emphasizing essential elements within their compositions. One crucial difference between this approach and other „rules” of composition lies in the unconscious nature of placing details along these lines, in contrast to consciously applying rules like the Rule of Thirds for subject placement. Additionally, photographers employ the Diagonal Method to position essential details that contribute to the narrative or hold personal significance in

psychological or emotional terms rather than including elements of minimal importance, such as streetlamps or the horizon.

Conclusion

While smaller part of high-school students finds mathematics enjoyable and rewarding, significant part considers mathematics difficult because many of its concepts are abstract and require regular and continuous learning to be comprehensively understood. Especially nowadays, when the students are raised in digital era and their attention time is low, they prefer visual form and demand fast response. In contrary, the mathematics instructional approaches can not abandon some traditional methods, for the mathematics to be captivated with its strictly determined rules, order, structure, systematization and harmony. The conventional strategies and methods employed in mathematics education (ex-cathedra teaching, full lecture handwritten on board, regular homeworks, tests that necessitate complete handwritten solutions) appear insufficient for the Z-generation and even more so for the Alpha generation of students. They are perceived as old-fashioned, ineffective, with lack of practical approaches and connections to the real world, while the expectations are related to digital literacy, experiential learning, interactivity, visual forms, and immediacy. Meeting these expectations, which come not only from students, but also from their parents, poses a significant challenge for mathematics teachers, particularly high-school teachers.

Real numbers represent any quantity that can be found in the real world, which makes the referring high-school lesson fundamental in science. They are defined as disjoint union of rational and irrational numbers. Thus, neglecting the concept of irrational numbers results in an incomplete understanding of real numbers. Yet often fallacies are observed related to the knowledge of irrational numbers. Misconceptions are detected in relation to: stating the definition of an irrational number, detection whether a given number is irrational or not, mixing rational approximation of an irrational number with the number itself, correct classification of a list of given numbers, and placement on the real axis. An important link with the rational numbers can be made by acknowledging the rational approximations of an irrational numbers. Having made this clear, will enable connecting the irrational numbers with practical applications and making the lesson real-life relevant. In this paper we elaborate the application of a representative of irrational numbers, namely the Golden Ratio, or more precisely, its rational approximation, which defines the Golden Ratio principle.

The Golden Ratio is a powerful mathematical principle artists have employed throughout history to create aesthetically pleasing and balanced compositions. It has found its way into iconic artworks, including architectural structures like the Egyptian pyramids and the Parthenon, as well as paintings by Leonardo da Vinci

or the designs of Le Corbusier. The Golden Ratio also provides a basis for some of the most fundamental compositional techniques in lens-based media, enabling artists to harmoniously divide the picture space and arrange the compositional elements in relation to one another and the whole. The strategic placement of key elements at Golden Points, determined by the Phi Grid and aligned with the Golden Ratio, enhances balance and harmony in compositions, with cinematographers often using this technique to position the human face, particularly the eyes, in alignment with the Golden Ratio between the hairline and the chin. With its logarithmic growth pattern, the Golden Spiral can also be utilized as a valuable tool in lens-based media to create visually engaging images, guide the viewer's eye through the frame, position key elements, and enhance aesthetic appeal. The Rule of Thirds suggests dividing the screen into three equal columns and rows with four lines, with areas of interest ideally falling near the lines' intersections. The Diagonal Method helps artists position essential details along the bisection lines formed by dividing corners into equal 45-degree angles, mainly used for narrative or emotional significance. Ultimately, achieving a successful composition involves a balance between adhering to guidelines, evaluating relationships between elements, and employing an intuitive approach to composition.

A mathematical class enriched with beautiful art works will should bring freshness and beautiful emotions in class, making the mathematical lecture pleasing, tangible, meaningful and appropriate for the newest generations of students. Such lesson followed by explanation why this connection is made, will create active learning environment and may inspire students not only to better understand the mathematical matter, but also to develop divergent thinking and creativity. It can be presented in a regular or additional mathematics class, to support interdisciplinarity, crossdisciplinarity or integrated teaching.

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